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Recently developed chiral nucleon–nucleon (NN) forces at next-to-leading order (NLO) that describe NN phase shifts up to about 100 MeV fairly well have been applied to 3N and 4N systems. Faddeev-Yakubovsky equations have been solved rigorously. The chiral NLO forces depend on a momentum cut-off Λ lying between 540-600 MeV/c. The resulting 3N and 4N binding energies are in the same range as found using standard NN potentials. In additon, low–energy 3N scattering observables are very well reproduced like for standard NN forces. Surprisingly, the long standing A_y–puzzle is resolved at NLO. The cut-off dependence of the scattering observables is rather mild.

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Effective field theory (EFT) has become a standard tool in modern physics and is applied to a large variety of systems. It can also be used to construct nuclear forces in a systematic and controlled manner. The spontaneously and explicitely broken chiral symmetry of QCD can be implemented in the EFT formulated in terms of the asymptotically observed Goldstone boson (pion) and matter (nucleon) fields. In the purely pionic and the pion-nucleon systems, there is an expansion parameter which is a typical external momentum (or the quark masses) divided by a hadronic mass scale of the order of the ρ meson or the nucleon mass. Any S-matrix element or transition current can be systematically expanded in terms of this small parameter based on a systematic power counting. In systems with more than one nucleon, an additional non-perturbative resummation is mandatory to deal with the shallow nuclear bound states. This idea was put forward by Weinberg [1] and taken up by van Kolck and collaborators [2,3] in the construction of two- (NN) and three-nucleon (3N) forces. One basically constructs a potential based on the power counting and calculates bound and scattering states by use of a properly regularized Lippmann-Schwinger or Schrödinger equation. One outstanding result was that 3N forces (3NF) vanish to leading order [1]. Other groups also investigated low energy properties of NN systems along these lines [4,5]. A different counting scheme was proposed by Kaplan et al. [6] (KSW) working directly with the scattering amplitudes instead with an effective Hamiltonian like it is the case along the lines proposed by Weinberg. Another important feature which distinguishes the KSW approach from Weinberg's is the perturbative treatment of the one-pion exchange. Independent of these differences, such type of framework for the first time offers the possibility of calculating nuclear forces directly from fundamental principles and has a direct link to the chiral properties of QCD. Furthermore, this approach is firmly based on quantum field theory and avoids ill-defined concepts like meson-nucleon form factors.

In [7] we have taken up Weinberg's idea and constructed a NN and 3N potential based on the most general effective chiral pion-nucleon Lagrangian using the method of unitary transformations. In this method the field theoretical pion-nucleon Hamiltonian is decoupled such that an effective purely nucleonic Hamiltonian consistent with a power counting scheme arises. Previous results were obtained in time-ordered perturbation theory, which lead to energy-dependent nuclear forces. In our formalism, we arrive at hermitian energy-independent nuclear forces which we consider to be a major advantage with respect to applications to 3N and 4N systems, the issue of this letter. In [1,3,7,8] NN forces have been developed at leading, next-to-leading and next-to-next-to-leading orders, LO, NLO and NNLO, respectively. At LO the potential is represented by the ordinary one-pion exchange (with a point-like coupling) as well as two contact interactions without derivatives. At NLO one includes the leading chiral two-pion exchange as well as all possible contact interactions with two derivatives, whereas at NNLO we have additional two-pion exchange with low-energy constants (LECs) determined from pion-nucleon scattering [9]. The forces are properly renormalized and contain nine parameters related to those four-fermion contact terms. The one- and two-pion exchange pieces are parameter free. The nine LECs have been uniquely fixed to low energy NN phase shifts in the s- and p-waves. The parameter free predictions for higher energies and partial waves and also deuteron properties are in general rather good. It was also observed that the NNLO predictions are better than the ones based on the NLO potential, as expected in a systematic EFT.

The natural question arises now, whether the NN forces based on chiral perturbation theory (χPT) will be also successful in describing 3N and 4N low-energy observables. To that aim we solve the Faddeev-Yakubovsky equations rigorously for 3N and 4N systems [10–12] and determine binding energies and various scattering observables. To the best of our knowledge this is the first time

TABLE I. Neutron-proton phase shifts in our approach (upper row) compared to the Nijmegen PSA (middle row) and the CD-Bonn potential (lower row).

	1 MeV	5 MeV	10 MeV	20 MeV
	62.044	63.869	60.28	53.76
$^1S_0$	62.078	63.645	59.97	53.56
	62.069	63.627	59.96	53.57
	147.695	118.308	102.87	86.33
$^3S_1$	147.748	118.175	102.60	86.09
	147.747	118.178	102.61	86.12
	0.107	0.679	1.17	1.65
ϵ_1	0.105	0.674	1.16	1.66
	0.105	0.672	1.16	1.66
	-0.005	-0.181	-0.67	-2.07
$^3D_1$	-0.005	-0.184	-0.68	-2.06
	-0.005	-0.183	-0.68	-2.05
	-0.187	-1.493	-3.08	-5.54
$^1P_1$	-0.189	-1.503	-3.08	-5.47
	-0.187	-1.487	-3.04	-5.40
	0.187	1.676	3.73	7.06
$^3P_0$	0.177	1.608	3.62	6.92
	0.180	1.626	3.65	6.95
	-0.117	-0.994	-2.16	-4.18
$^3P_1$	-0.108	-0.932	-2.05	-4.01
	-0.108	-0.937	-2.06	-4.03
	0.020	0.237	0.70	2.05
$^3P_2$	0.022	0.255	0.72	1.84
	0.022	0.251	0.71	1.84

that χPT has been practically applied to nuclear systems beyond $A = 2$ within the Hamiltonian approach.

In this first application we restrict ourselves to the NLO NN potential. In a forthcoming article we shall go on to NNLO and include also 3NFs, which occur at that order the first time. The NLO results presented here are therefore parameter free and can serve as a good testing ground for the usefulness of the approach. Of course, some aspects of the 3N system have already been studied in nuclear EFT [13,14], but not as direct extensions of the NN system as done here.

In order to use the chiral NN forces in the NN Lippmann-Schwinger equation one has to introduce a momentum regulator Λ. We remark that this regularization on the level of the scattering equation is completely different from standard methods which are applied to individual diagrams. Here we use the smooth regulator, its precise form is given in [8]. In order to investigate the cut-off dependence of 3N and 4N observables we have generated several NN potentials corresponding to different cut-offs between Λ =540 and 600 MeV/c. They were all fitted to the $^1S_0$, $^3S_1$-$^3D_1$, $^1P_1$ and $^3P_{0,1,2}$ NN phase shifts up to $E_{\rm lab} = 100\,$MeV (for the potential parameters contact E.E.). In [8] we had already demonstrated that going to higher order in the EFT reduces the cutoff dependence and allows to choose larger values for the cut-offs, as expected from general arguments [15]. The resulting phase shifts for NLO are compared to the Nijmegen phase shift analysis [16] and the ones obtained from the CD-Bonn potential [17] in Table I. The agreement is fairly well and we know from [8] that one has to go to NNLO to improve systematically on that. Also, we restrict ourselves to the isospin symmetric case. Thus we do not take into account various charge independence and charge symmetry breaking effects like e.g. the pion mass difference, which are known to be significant at very low energies. Such effects can also be included systematically in our EFT [18].

TABLE II. Theoretical ^{3}H and ^{4}He binding energies for different cut-offs Λ compared to CD-Bonn predictions and to the experimental ^{3}H binding energy and the Coulomb corrected ^{4}He binding energy. The kinetic energies and S', P and D state probabilities for ^{4}He are also shown.

Potential	E_T [MeV]	$E_{^4\mathrm{He}}$	T [MeV]	S' [%]	P [%]	D [%]
NLO, 540	-8.284	-28.03	65.2	0.62	0.08	6.00
NLO, 560	-8.091	-26.91	68.2	0.68	0.09	6.41
NLO, 580	-7.847	-25.55	72.2	0.76	0.10	6.84
NLO, 600	-7.546	-23.96	77.7	0.86	0.11	7.30
CD-Bonn	-8.012	-27.05	77.6	0.48	0.22	10.72
exp	-8.48	-29.00	—	—	—	—

Let us regard 3N and 4N binding energies now. In addition to the relative motion in the NN subsystem there occur relative motions of the third and fourth nucleon now. In the spirit of a low momentum theory one has to expect that those additional relative momenta should be also small. This turns out to be true as the numerics tells us. For standard NN potentials, which are defined for all momenta up to infinity, one introduces a momentum cut–off for the numerical integrations. Since the pertinent integrals are convergent, the results are cut–off independent. Using the chiral NN potential we could reduce the cut-offs in those relative momenta from their typical values 8 fm^{-1} arising with standard potentials down to 4 fm^{-1} without changing the result. In the Faddeev-Yakubovsky equations there is no mechanism which could introduce high momenta if the NN subsystem momenta are small to start with. We find for the fully converged solutions the 3N and 4N binding energies as given in Table II. The ranges are compatible with what is found using realistic potentials [10]. Note that the NN forces were included up to the total NN angular momentum of $j_{\rm max}$=6.

Also 3N scattering can be solved rigorously nowadays [19] and we show in Figs. 1, 2 a small selection out of the great wealth of observables in comparison to data and the theoretical predictions of CD-Bonn. Three energies, one below the *nd* break-up threshold and two above are chosen. Note further that we do not indicate error bars for the data since in most cases they will be not

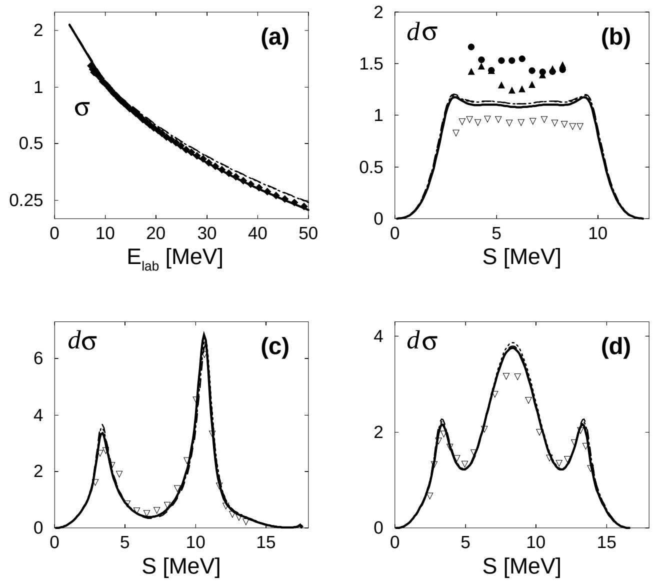

FIG. 1. (a): The *nd*- total cross section (in barn) for the chiral forces ($\Lambda = 540$ MeV/c, dotted curve; $\Lambda = 600$ MeV/c, dashed curve), and CD-Bonn (thick solid curve). (b), (c) and (d): Chiral NN force and CD-Bonn predictions for *nd* break-up cross section $d\sigma \equiv \frac{d^5\sigma}{d\Omega_1 d\Omega_2 dS}$ $[\frac{\text{mb}}{\text{sr}^2\text{MeV}}]$ at E_{lab} =13 MeV along the kinematical locus S. The various break-up configurations are chosen according to Figs. 42, 39 and 35 in [19], respectively. *pd* data are (▽) [20]; *nd* data are (▲) [21], (●) [22], (♦) [23].

distinguishable on this scale. In all cases we show the predictions of the chiral NN potentials for the cut-offs $\Lambda = 540$ and $\Lambda = 600$ MeV/c and compare it to the prediction of one of the most modern, so–called realistic NN potentials, the CD-Bonn [17]. As the simplest observable we show first the *nd* total cross section in Fig. 1(a). The three theoretical curves overlap completely at very low energies and then the chiral predictions start to deviate somewhat from the data as expected for our EFT at NLO.

In case of the 3N break-up we selected in Fig. 1(b,c,d) a few often investigated configurations, the space-star, a final state interaction peak configuration and a quasi–free scattering (QFS) configuration, respectively. We find very good agreement of the chiral NN force predictions with the one from CD-Bonn. In case of QFS (d) and space-star (b) some of the discrepancies are expected to be caused by Coulomb force effects not included in the theory. The upper group of data in (b) are *nd* data and the disagreement with the theory presents a well known puzzle at the moment [24].

For elastic *nd* scattering we display in Fig. 2 the angular distributions, the nucleon analyzing power A_y and the tensor analyzing powers T_{2k}. Except for A_y there are no *nd* data for those energies. The discrepancies be-

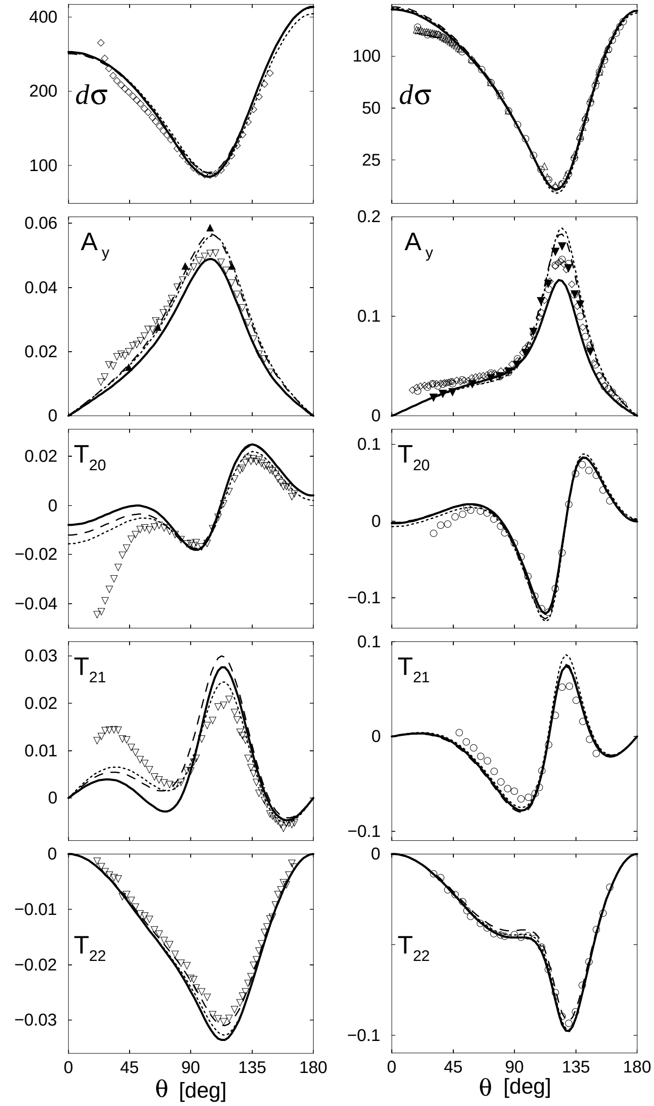

FIG. 2. *nd* elastic scattering observables at $E_n = 3$MeV (left column) and $E_n = 10$MeV (right column). *pd* data are (◇) [25], (▽) [26], (△) [27], (○) [28]. *nd* data are (▲) [29], (▼) [30]. For remaining notations see Fig. 1.

tween data and theory for T_{2k} and for the differential cross section can be traced back to *pp* Coulomb force effects [31]. Thus except for A_y the agreement of CD-Bonn (thick solid curve) with the data is rather good, which is a well known fact and is just given for the sake of orientation. The dotted and dashed curves are the chiral NN force predictions for $\Lambda = 540$ and $\Lambda = 600$ MeV/c, respectively. The $d\sigma$ and T_{2k} agree rather well with the CD-Bonn result and thus with the data. We consider this to be an important result, demonstrating that the chiral NN forces are very well suited to also describe low-energy 3N scattering observables rather quantitatively. On top of that, surprisingly for us, the chiral force predictions are now significantly higher in the maxima of A_y than

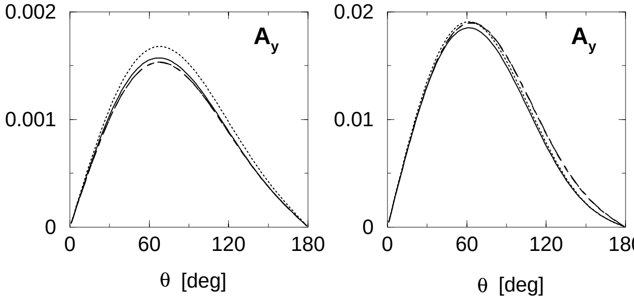

FIG. 3. np analyzing powers A_y at 3 (left panel) and 12 MeV (right panel) for the chiral forces ($\Lambda = 540$ MeV/c, dashed curve; $\Lambda = 600$ MeV/c, dot-dashed curve), PSA (solid curve) and CD-Bonn (dotted curve).

for CD-Bonn and break the long standing situation that all standard realistic NN forces up to now underpredict the maxima by about 30 %. This is called the A_y–puzzle [19]. We are now in fact rather close to the experimental *nd* values. Since we restrict ourselves to NLO we can not expect a final answer from the point of view of chiral dynamics but this result for A_y is very interesting. In that context it is important to note that on a 2N level the chiral potential predictions agree well with the predictions based in the Nijmegen PSA as is shown in Fig. 3. There we also include the CD-Bonn predictions. This agreement is especially important for the *np* analyzing power A_y, since it is rather sensitive to the ^3P_j NN phase shifts, which influence also strongly the 3N A_y.

These very first results using chiral NN forces in 3N and 4N systems are very promising. Since we restricted ourselves to NLO we could not expect a very good description of the data, since on-shell properties are not perfectly well fitted. But the results show that these effective chiral forces are very well suited to describe also low energy properties of nuclear systems beyond $A = 2$. They agree rather well with standard nuclear force predictions as exemplified with CD-Bonn and most importantly they break the stagnation in the A_y puzzle. Our result provides a counter example to the suggestion [33] that NN forces alone can not describe 2N and 3N A_y's at the same time and 3NF's should cure the 3N A_y puzzle. Examples for such trials can be found in [31,32].

It will be very interesting to perform the next step and use the NNLO NN forces, which is a systematic improvement. On this level also 3N forces have to be incorporated the first time, which in χPT are defined consistently with the NN force. It should be mentioned further that due to the underlying Lagrangian the coupling of external probes (photons for instance) is well defined and exchange currents consistent to the nuclear forces can be generated. Also the hybrid approaches [34,35] can be avoided and wavefunctions based on the chiral dynamics can be used directly instead of wavefunctions generated by standard NN forces. This appears advisable since in this study for NLO we noticed that kinetic energies in 3N and 4N bound states are smaller and two-nucleon correlation functions are smoother than for standard NN forces, indicating a change of the wave functions.

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